AN EXPRESS TAXI (Lithuania)

At first sight it might be very tempting to apply the *Greedy method* for solving this problem. Suppose that we calculate the price of the journey per one kilometer for each route. Then choose the cheapest route, take this bus and go up to the length of the route becomes longer than the remaining distance. Then find the second cheapest route and repeat everything until the passenger drives the required number of kilometers.

However, the following input data prove that the *Greedy approach* cannot be applied successfully:

Distance	1	2	3	4	5	6	7	8	9	10
Cost of driving	10	10	30	5	5	60	70	80	90	100

Suppose that the passenger has to go 17 kilometers. We will get the least costly route for driving 1 km if we choose the taxi going 5 km without a stop. Having taken this taxi 3 times, there are 2 kilometers left afterwards. The passenger goes those two kilometers without a change. So, the total costs of the journey is

$$3 \times 5 + 10 = 25$$

However, it is possible to find a cheaper route. Let us take the taxi driving 4 km 3 times and the taxi driving 5 km once. The total cost would be:

$$3 \times 5 + 1 \times 5 = 20$$

To solve the problem we will use *the Dynamic method*.

We know the cheapest way to go 1 km (there is simply no other choice in this case).

The passenger can go 2 km in two ways: either to take the taxi which drives 2 km without a stop once or the taxi going 1 km twice. The smaller of the two prices will be the cheapest for going 2 kilometers.

Now, suppose that we know the cheapest routes to go 1, 2, 3... $k - 1$ kilometers. How to find the cheapest route for going k kilometers?

There are 10 (or less if $k \le 9$) ways to go k kilometers: after going $k - 10$ (or $k - 9$, $k - 8$,... $k - 1$) in the cheapest way, the passenger changes taxis and goes 10 (or 9, 8, 7, ... 1) kilometers without a stop.

The cheapest of the 10 analysed routes will be the searched route. Let us make an additional table with n columns to trace the route. The k-th column will show the cheapest price for going k kilometers and the number of kilometers that have to be gone without a stop after the last change.

Here is the completed table for the data above:

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
costs	0	10	10	20	5	5	15	15	10	10	10	20	15	15	15	15	20	20
km	0	1	2	2	4	5	5	5	4	5	5	5	4	5	5	5	4	5

The arrows help to trace the cheapest route.

There might be more than one correct answers to the input data of this problem. Yet, in the floppy you will find only one correct result.